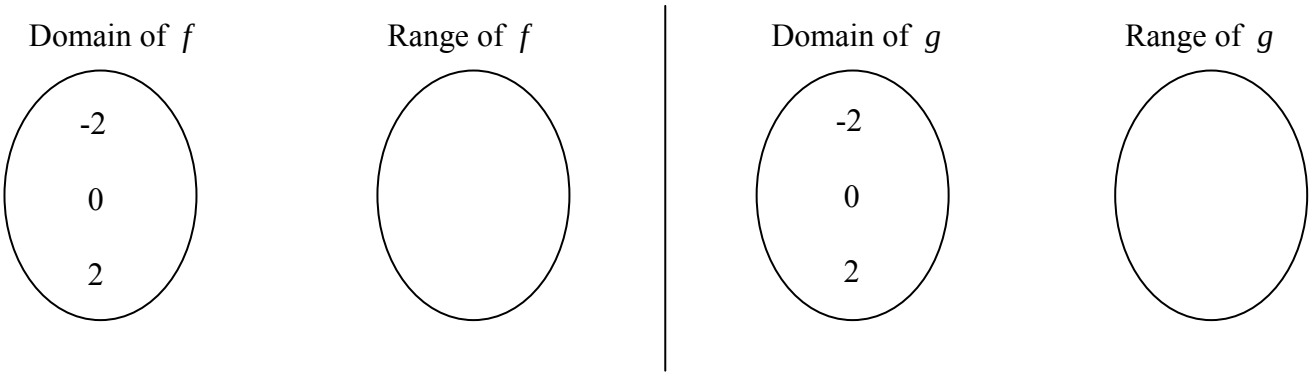
Name: _____ Date: _____

ONE-TO-ONE FUNCTIONS
COMMON CORE ALGEBRA II

Functions as rules can be divided into various categories based on shared characteristics. One category is comprised of functions known as one-to-one. The following exercise will be illustrate the difference between a function that is one-to-one and one that is not.

***Exercise* #1:** Consider the two simple functions given by the equations $f(x) = 2x$ and $g(x) = x^2$.

(a) Map the domain $\{-2, 0, 2\}$ using each function. Fill in the range and show the mapping arrows.



(b) What is fundamentally different between these two functions in terms of how the elements of this domain get mapped to the elements of the range?

ONE-TO-ONE FUNCTIONS

A function $f(x)$ is called one-to-one if $x_1 \neq x_2$ implies that $f(x_1) \neq f(x_2)$.

(In other words, different inputs give different outputs.)

***Exercise* #2:** Of the four tables below, one represents a relationship where *y* is a one-to-one function of *x*. Determine which it is and explain why the others are not.

(1)

x	*y*
4	2
4	-2
9	3
9	-3

(2)

x	*y*
-2	1
-1	0
0	1
1	2

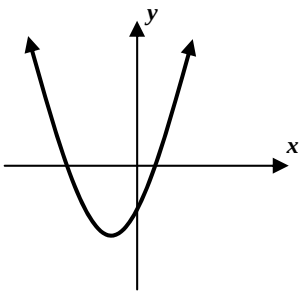
(3)

x	*y*
1	2
2	4
3	8
4	16

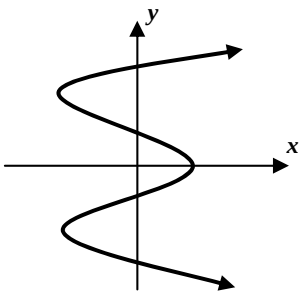
(4)

x	*y*
-3	10
-2	9
-1	7
-2	10

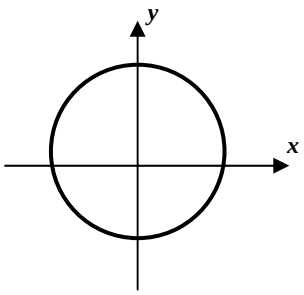
***Exercise* #3:** Consider the following four graphs which show a relationship between the variables *y* and *x*.



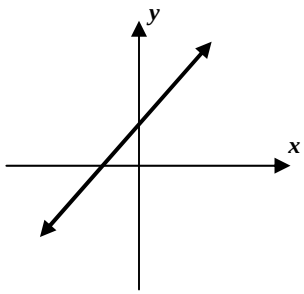
(1)



(2)



(3)



(4)

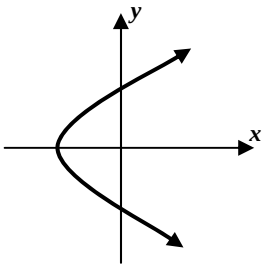
(a) Circle the two graphs above that are functions. Explain how you know they are functions.

(b) Of the two graphs you circled, which is one-to-one? Explain how you can tell from its graph.

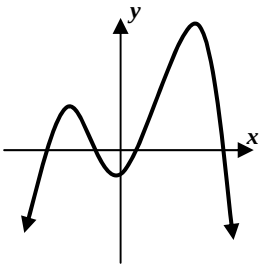
THE HORIZONTAL LINE TEST

If any given horizontal line passes through the graph of a function at most one time, then that function is one-to-one. This test works because horizontal lines represent constant *y*-values; hence, if a horizontal line intersects a graph more than once, an output has been repeated.

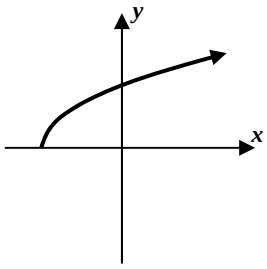
***Exercise* #4:** Which of the following represents the graph of a one-to-one function?



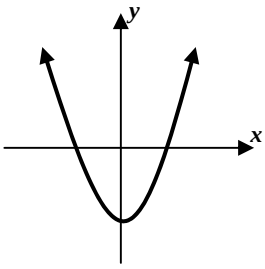
(1)



(2)



(3)



(4)

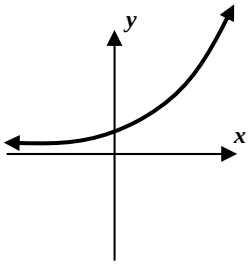
***Exercise* #5:** The distance that a number, *x*, lies from the number 5 on a one-dimensional number line is given by the function $D(x)=|x-5|$. Show by example that $D(x)$ is *not* a one-to-one function.

Name: _____ Date: _____

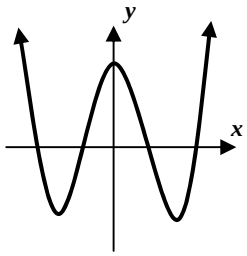
ONE-TO-ONE FUNCTIONS
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

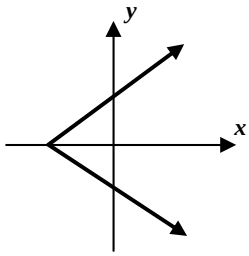
1. Which of the following graphs illustrates a one-to-one relationship?



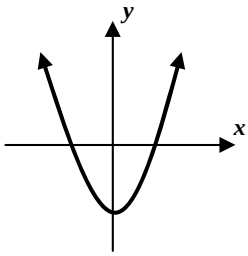
(1)



(2)



(3)



(4)

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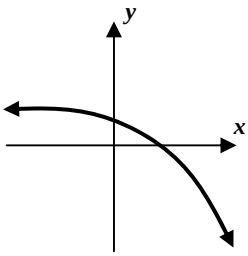
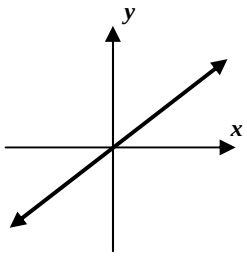
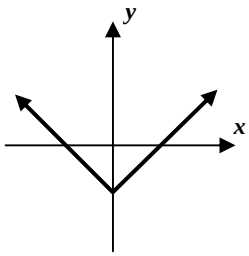
2. Which of the following graphs does *not* represent that of a one-to-one function?



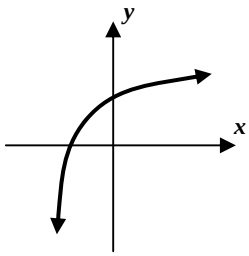
(1)



(2)



(3)



(4)

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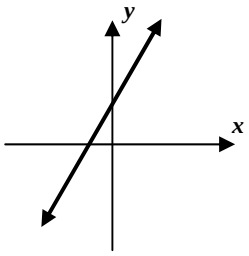
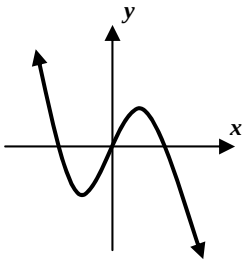
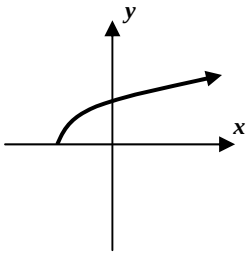
3. In which of the following graphs is each input *not* paired with a *unique* output?



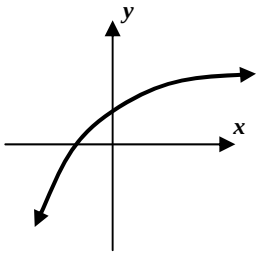
(1)



(2)



(3)



(4)

_____

4. In which of the following formulas is the variable y a one-to-one function of the variable x? (Hint – try generating some values either in your head or using TABLES on your calculator.)

(1) $y = x^2$ (3) $y = 2x$

(2) $y = |x|$ (4) $y = 5$

_____

5. Which of the following tables illustrates a relationship in which y is a one-to-one function of x?

(1)

x	y
-2	-1
0	-3
2	-1
4	1
6	3

(2)

x	y
-2	-8
-1	-1
0	0
1	1
2	8

(3)

x	y
-2	-5
-1	-4
0	-1
-1	7
-2	5

(4)

x	y
-2	11
-1	-4
0	-5
1	-4
2	11

_____

APPLICATIONS

6. A recent newspaper gave temperature data for various days of the week in table format. In which of the tables below is the reported temperature a one-to-one function of the day of the week?

(1)

x	y
Mon	75
Tue	68
Wed	65
Thu	74

(2)

x	y
Mon	75
Tue	72
Wed	68
Thu	72

(3)

x	y
Mon	58
Tue	52
Mon	81
Tue	76

(4)

x	y
Mon	56
Tue	58
Mon	85
Tue	85

_____

7. Physics students drop a basketball from 5 feet above the ground and its height is measured each tenth of a second until it stops bouncing. The height of the basketball, h, is clearly a function of the time, t, since it was dropped.

(a) Sketch the general graph of what you believe this function would look like.

(b) Is the height of the ball a one-to-one function of time? Explain your answer.

REASONING

8. Consider the function $f(x) = \text{round}(x)$, which rounds the input, x, to the nearest integer. Is this function one-to-one? Explain or justify your answer.